Lecture 4: Fundamentals Part 3
Open-Channel Flow 1

WMD651: Water Resources Systems Design

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Open-Channel Flow

- Pressurized and open-channel flow are fundamentally different
- Water exposed to atmospheric pressure at surface
- Consequently, **water surface = HGL**

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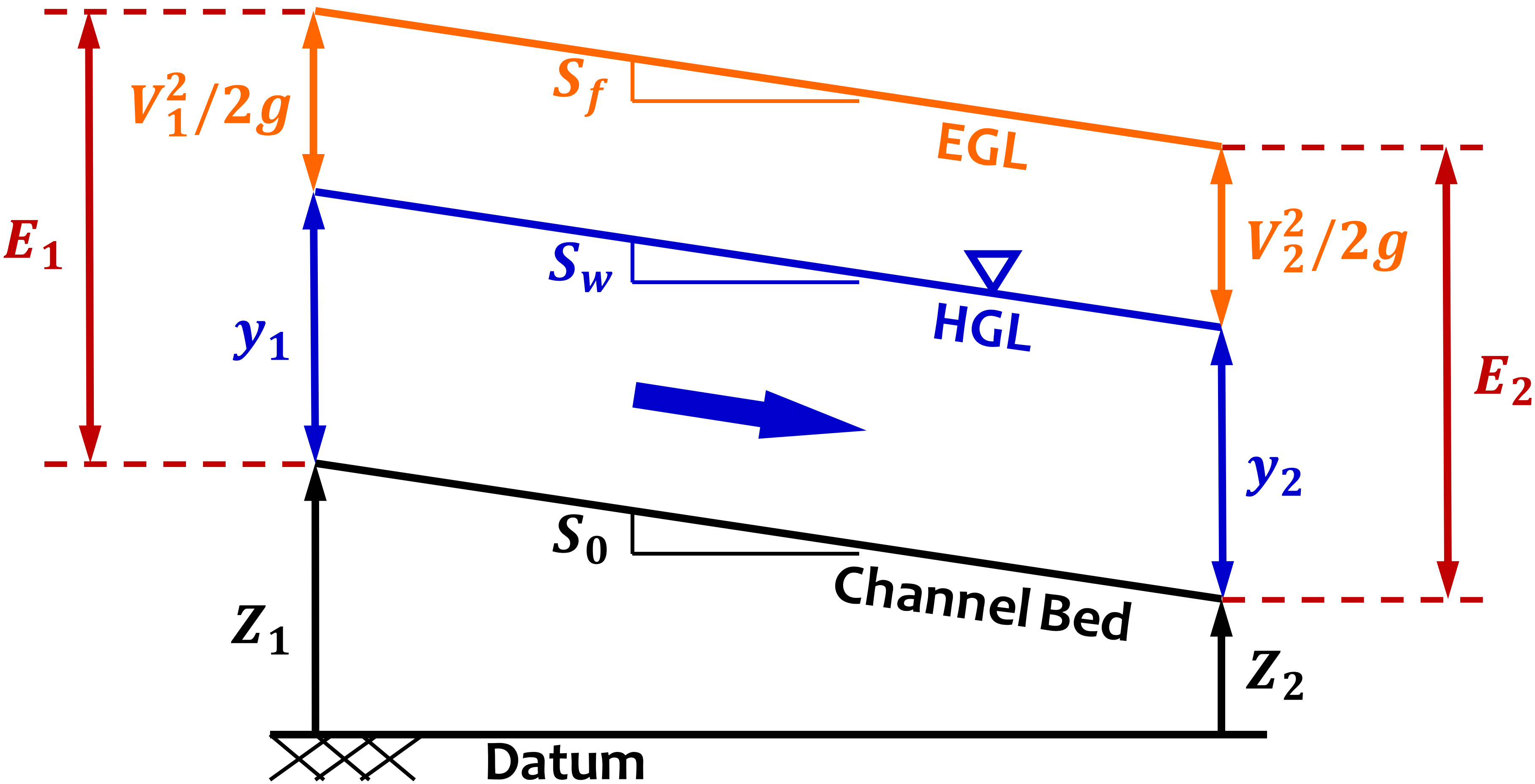
Tracom Fiberglass Products, 2019

HGL and EGL for Open-Channel Flow



Characteristics of Open Channels

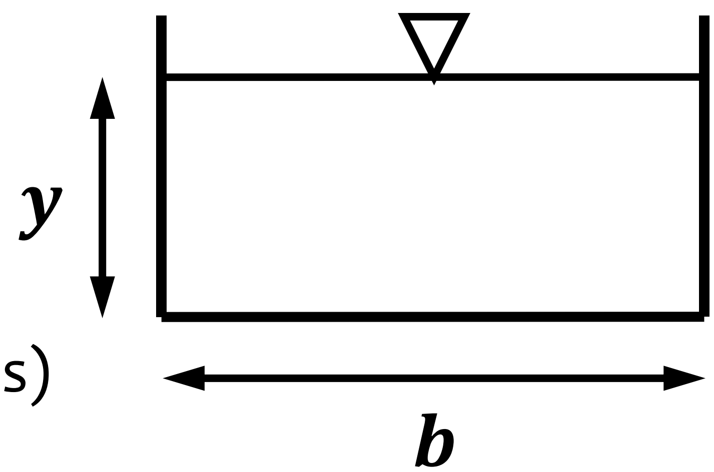
- Applications:
  - Rivers, canals, and streams
  - Overland flow (drainage)
  - Sewers (sanitary, storm, combined)
  - Culverts
  - Agricultural irrigation
- Slope terms:
  - Friction (EGL) slope: $S_f = (E_2 - E_1)/L$
  - Water level (HGL) slope: $S_w = (y_2 - y_1)/L$
  - Bed slope: $S_0 = (Z_2 - Z_1)/L$
- Slope terms are not always equal → **depends on flow conditions**

Characteristics of Open Channels

- Specific energy:
  - Equation: $E = y + V^2/2g$
  - Represents unit energy relative to channel bed
- Specific momentum:
  - Equation: $M = A\bar{y} + V^2/g$
  - $\bar{y}$ = distance from water surface to area centroid
  - Represents unit momentum of flow
- Flow conditions:
  - Temporal: steady or unsteady
  - Spatial: uniform, gradually varied, rapidly varied

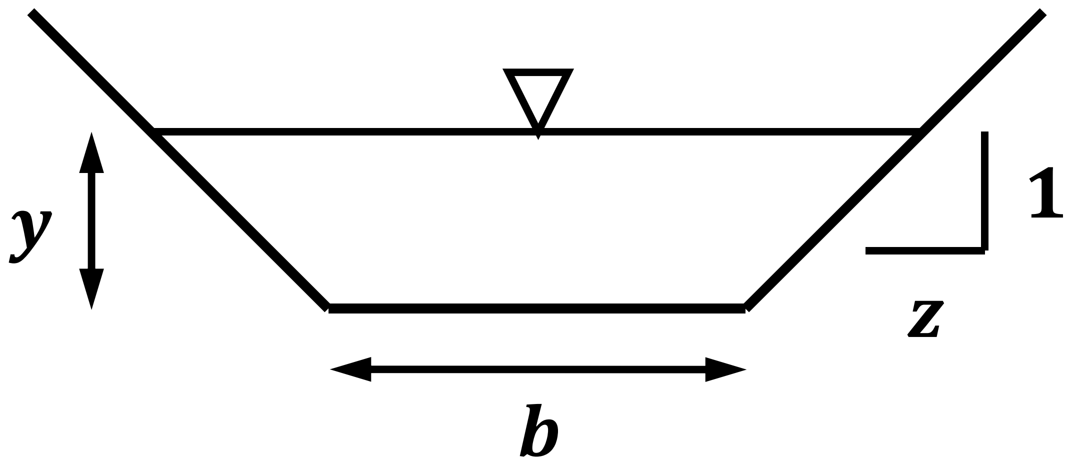
Continuity Equation and Geometry

- Continuity relates flow, flow area, and mean velocity: $\boldsymbol{Q = VA}$
- Area depends on channel shape and depth (y)
- Many channel shapes are possible, even compound shapes
- **Rectangular channels:**
  - $A = by$
  - $P = b + 2y$ (wetted perimeter)
  - $R = \frac{A}{P} = \frac{by}{b+2y}$ (hydraulic radius)



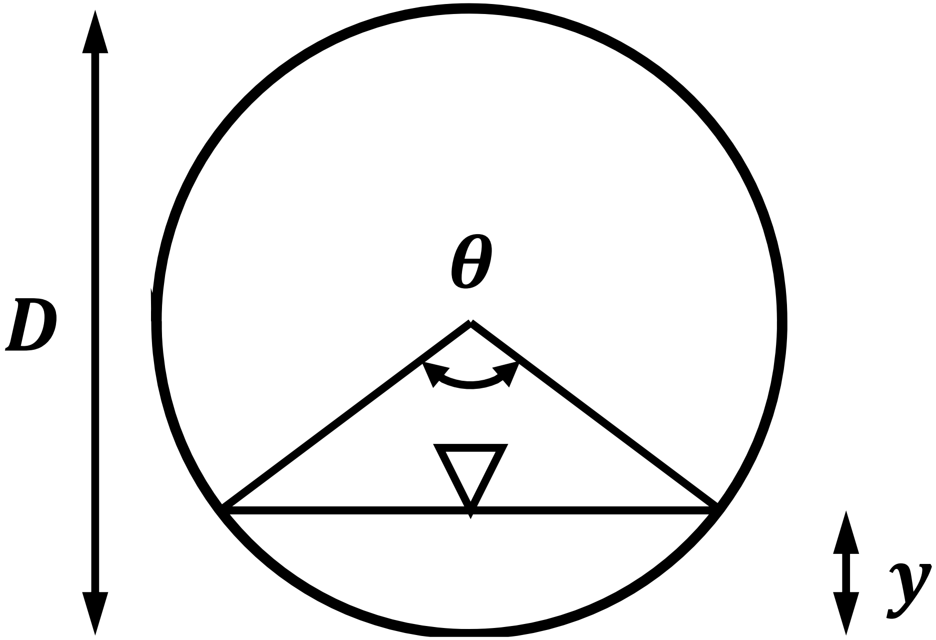
Channel Shapes and Geometry

- Trapezoidal channels:
  - $A = y(b + zy)$
  - $P = b + 2y\sqrt{1 + z^2}$
  - $R = \frac{y(b+zy)}{b+2y\sqrt{1+z^2}}$



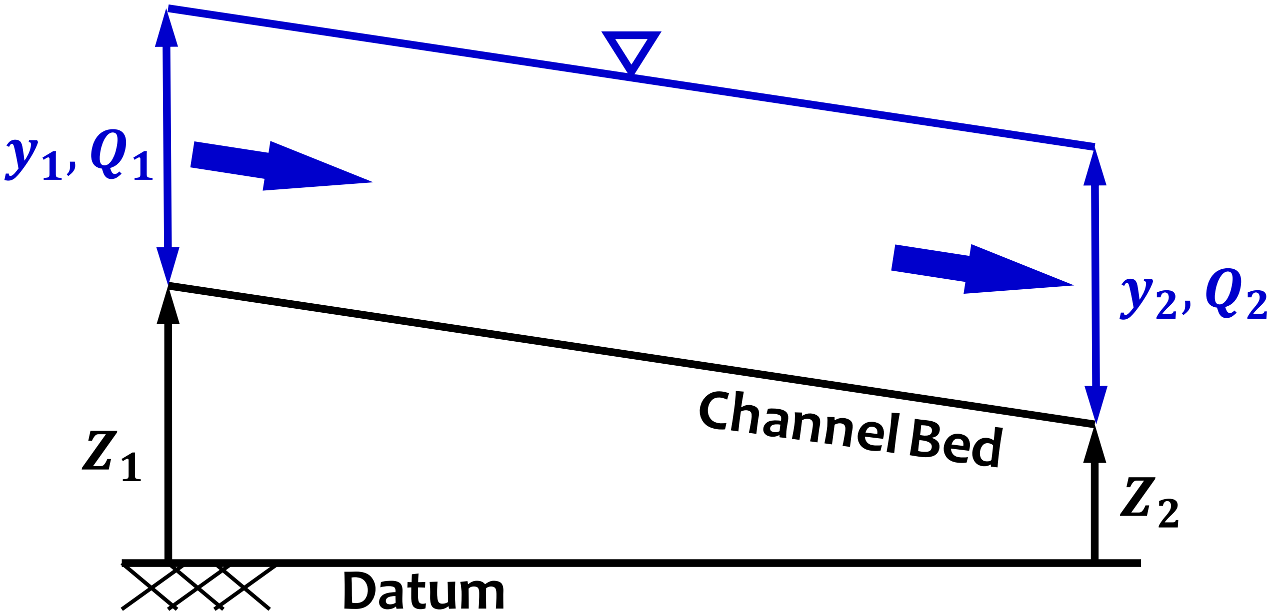
Channel Shapes and Geometry

- **Circular channels:**
  - $y = \frac{1}{2}D[\,1 - \cos(\theta/2)]$
  - $A = \frac{1}{8}D^2(\theta - \sin\theta)$
  - $P = \frac{1}{2}D\theta$
  - $R = \frac{1}{4}D\left(1 - \frac{\sin\theta}{\theta}\right)$
- Note: θ is in [radians]



Uniform Flow

- Consider the channel section shown below:



- If $y_1 = y_2$ and $Q_1 = Q_2$, we call this uniform flow
- Uniform flow is defined by $\boldsymbol{S_f = S_w = S_0}$
- Corresponding depth known as the normal depth (y_n)

Manning Equation

- Describes relationship between head losses (S_f) and mean velocity for open-channel flow
- Manning equation: $\boldsymbol{V = \frac{1}{n} R^{2/3} S_f^{1/2}}$
  - V = flow velocity (m/s)
  - n = Manning roughness (typ. n = 0.010 to 0.025)
    - Concrete pipe: typical n = 0.010 to 0.030
    - Natural channels: typical n = 0.050 to 0.10
  - $R = A/P$ is the hydraulic radius (m)
  - A = flow area (m^2)
  - P = wetted perimeter (m)
  - S_f = friction slope (e.g., 0.01)
- From continuity: $\boldsymbol{Q = \frac{1}{n} A R^{2/3} S_f^{0.5}}$ (m^3/s)

Manning Equation and Uniform Flow

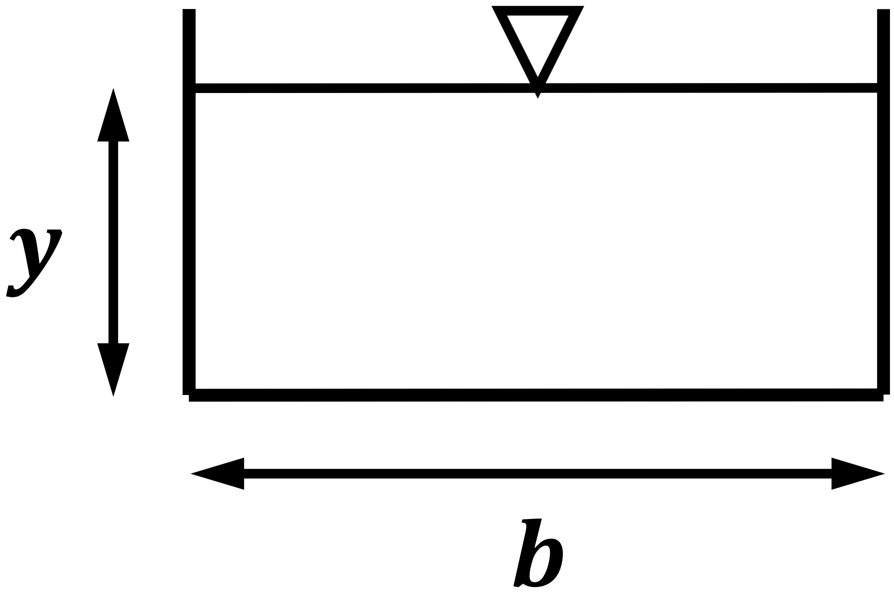
- $S_f = S_0$ for uniform flow
- Thus, for uniform flow, the Manning equation is:

$$\boldsymbol{Q = \frac{1}{n} A R^{2/3} S_0^{1/2}}$$

- The set of y_n, A, and R that satisfy the Manning equation represents uniform flow conditions
- Often, we need to determine y_n for a channel with a given flow

Example 1 – Normal Depth

- **Problem:** Determine the normal depth y_n for a rectangular channel with b = 0.90 m, Q = 0.75 m^3/s, n = 0.025, and S_0 = 1.5%.



Example 1 – Normal Depth

- **Solution:**
  - Calculate the normal depth by solving the Manning equation for y_n.
  - For a rectangular channel, $A = by$ and $R = \frac{by}{b+2y}$.
  - Manning equation for uniform flow: $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$
  - Solution approach:
    - Rearrange Manning eq. to group known terms
    - Estimate y_n and calculate A and R
    - Check whether estimate satisfies Manning equation
    - Update and recheck estimate as needed

Example 1 – Normal Depth

- **Solution (continued):**
  - Rearrange Manning equation to group known terms:
    - $Q = \frac{1}{n} AR^{2/3} S_f^{0.5}$
    - $\frac{nQ}{S_0^{0.5}} = AR^{2/3}$ → referred to as the section factor
  - Calculate left side of equation:
    - $\frac{nQ}{S_0^{0.5}} = 0.025(0.075\ m^3 s)(0.015)^{0.5}$
    - $= 0.153\ m^{8/3}$
  - We need y_n that satisfies $AR^{2/3} = 0.153\ m^{8/3}$
  - $AR^{2/3} = (by)\left(\frac{by}{b+2y}\right)^{2/3}$

Example 1 – Normal Depth

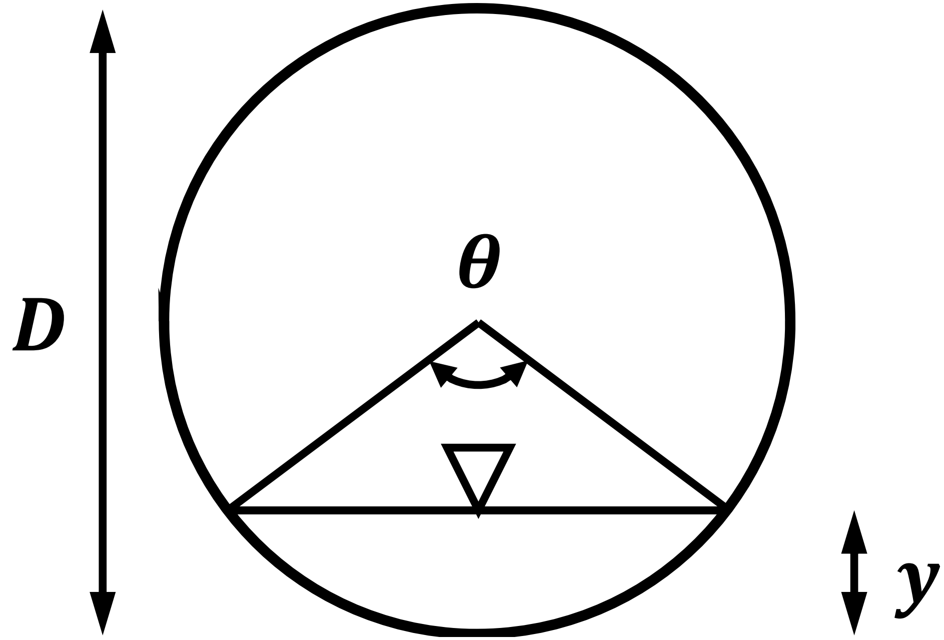
- **Solution (continued):**
  - Trial 1: guess y_n = 1.0 m
    - $A = by = (0.90\ m)(1.0\ m) = 0.90\ m^2$
    - $R = \frac{by}{b+2y} = \frac{(0.90\ m)(1.0\ m)}{(0.90\ m)+2(1.0\ m)} = 0.31\ m$
    - $AR^{2/3} = (0.90\ m^2)(0.31\ m)^{2/3} = 0.413\ m^{8/3} > 0.153\ m^{8/3}$
    - Thus trial y_n is too large → try smaller y_n

Example 1 – Normal Depth

- **Solution (continued):**
  - Trial 1: guess y_n = 1.0 m
    - $A = by = (0.90\ m)(1.0\ m) = 0.90\ m^2$
    - $R = \frac{by}{b+2y} = \frac{(0.90\ m)(1.0\ m)}{(0.90\ m)+2(1.0\ m)} = 0.31\ m$
    - $AR^{2/3} = (0.90\ m^2)(0.31\ m)^{2/3} = 0.413\ m^{8/3} > 0.153\ m^{8/3}$
    - Thus trial y_n is too large → try smaller y_n
  - Trial 2: guess y_n = 0.5 m
    - $A = by = (0.90\ m)(0.5\ m) = 0.45\ m^2$
    - $R = \frac{by}{b+2y} = \frac{(0.90\ m)(0.5\ m)}{(0.90\ m)+2(0.5\ m)} = 0.237\ m$
    - $AR^{2/3} = (0.90\ m^2)(0.237\ m)^{2/3} = 0.172\ m^{8/3} > 0.153\ m^{8/3}$
    - Closer, but trial y_n is still too large
  - If one continues iterative, we get y_n = 0.457 m

Example 2 – Flow Calculation

- **Problem:** A circular pipe has a diameter of D = 450 mm, a flow depth of y = 315 mm, a slope of S_0 = 0.01 m/m, and a Manning roughness of n = 0.013. Assuming uniform flow conditions, determine the flow rate.



Example 2 – Flow Calculation

- **Solution:**
  - For uniform flow, use Manning equation to calculate Q
  - First calculate θ for y_n = 0.315 m:

Example 2 – Flow Calculation

- **Solution:**
  - For uniform flow, use Manning equation to calculate Q
  - First calculate θ for y_n = 0.315 m:
    - For circular sections: $y = \frac{1}{2}D[\,1 - \cos(\theta/2)]$
    - Rearrange for θ:
      - $\frac{2y}{D} = \ 1 - \cos(\theta/2)$
      - → $\theta = 2 \cdot \cos^{-1}(1 - 2y/D)$
    - Solve for θ:
      - $\theta\ = 2 \cdot \cos^{-1}\left(1 - \frac{2y}{D}\right)$
      - $= 2 \cdot \cos^{-1}\left[1 - \frac{2(0.315\ m)}{(0.450\ m)}\right]$
      - $= 3.96\, rad$

Example 2 – Flow Calculation

- **Solution (continued):**
  - Calculate area, A:
    - $A = \frac{1}{8}D^2(\theta - \sin\theta) = \frac{1}{8}(0.450\ m)^2[3.96 - \sin(3.96)] = 0.119\ m^2$
  - Calculate hydraulic radius, R:
    - $R = \frac{1}{4}D\left(1 - \frac{\sin\theta}{\theta}\right) = \frac{1}{4}(0.45\ m)\left(1 - \frac{\sin(3.96)}{3.96}\right) = 0.133\ m$
  - Use the Manning equation to calculate flow, Q:
    - $Q = \frac{1}{n}AR^{2/3}S_0^{0.5}$
    - $= \frac{1}{0.013}(0.119\ m^2)(0.133\ m)^{2/3}(0.01\ m/m)^{0.5}$
    - $= 0.239\ m^3/s$

Example 2 – Flow Calculation

- **Alternative solution:**
  - As an alternative to the above, we can use reference tables such as that below:

Theta (rad)	y/D	a/A	r/R	q/Q
0.000	0.00	0.000	0.000	0.000
0.902	0.05	0.019	0.130	0.005
1.287	0.10	0.052	0.254	0.021
1.591	0.15	0.094	0.372	0.049
1.855	0.20	0.142	0.482	0.088
2.094	0.25	0.196	0.587	0.137
2.319	0.30	0.252	0.684	0.196
2.532	0.35	0.312	0.774	0.263
2.739	0.40	0.374	0.857	0.337
2.941	0.45	0.436	0.932	0.417
3.142	0.50	0.500	1.000	0.500

Theta (rad)	y/D	a/A	r/R	q/Q
3.342	0.55	0.564	1.060	0.586
3.544	0.60	0.626	1.111	0.672
3.751	0.65	0.688	1.153	0.756
3.965	0.70	0.748	1.185	0.837
4.189	0.75	0.804	1.207	0.912
4.429	0.80	0.858	1.217	0.977
4.692	0.85	0.906	1.213	1.030
4.996	0.90	0.948	1.192	1.066
5.381	0.95	0.981	1.146	1.075
6.271	1.00	1.000	1.002	1.001

Example 2 – Alternate Approach

- Upper case values represent those values for a pipe flowing full (i.e., y = D)
- a = relative area to total flow area
- q = relative flow to full flow at 100% depth
- r = relative hydraulic radius
- Full flow rate for circular conduit:
  - $Q_{full} = \frac{1}{n} A_{full} R_{full}^{2/3} S_0^{0.5}$
  - $A_{full} = \frac{1}{4} \pi D^2$
  - $R_{full} = \frac{1}{4} D$

Example 2 – Flow Calculation

- **Alternative solution (continued):**
  - First, calculate the relative depth:
    $$y/D = (315\ mm)/(450\ mm) = 0.70$$
  - From the table for y/D = 0.7: $q/Q_{full} = 0.837$
  - Calculate the full pipe flow (using $\theta = 2\pi\ rad$):
    - $A_{full} = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.450\ m)^2 = 0.159\ m^2$
    - $R_{full} = D/4 = (0.45\ m)/4 = 0.113\ m$
    - $Q_{full} = \frac{1}{n} A R^{2/3} S_0^{0.5}$
      $= \frac{1}{n}(0.159\ m^2)(0.113\ m)^{2/3}(0.01\ m/m)^{0.5}$
      $= 0.285\ m^3/s$
  - Determine actual flow:
    - $Q = \left(q/Q_{full}\right) \times Q_{full}$
    - $= 0.837 \cdot (0.285\ m^3/s) = 0.239\ m^3/s$